[2]Burnley, V. S., Swenson, G., and Culick, F. E. C., "Pulsed Instabilities in Combustion Chambers," *31st AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, San Diego, CA, AIAA, Washington, DC, 1995; also AIAA Paper 95-2430, 1995.

[3]Burnley, V. S., "Nonlinear Combustion Instabilities and Stochastic Sources," Ph.D. Dissertation, California Inst. of Technology, Pasadena, CA, 1996.

[4]Wicker, J. M., Greene, W. D., Kim, S.-I., and Yang, V., "Triggering of Longitudinal Combustion Instabilities in Rocket Motors: Nonlinear Combustion Response," *Journal of Propulsion and Power*, Vol. 12, No. 6, 1996, pp. 1148–1158.

[5]Jahnke, C. C., and Culick, F. E. C., "An Application of Dynamical Systems Theory to Nonlinear Combustion Instabilities," *Journal of Propulsion and Power*, Vol. 10, No. 4, 1994, pp. 508–517.

[6]Kim, S. I., "Nonlinear Combustion Instabilities in Combustion Chambers," Ph.D. Dissertation, Dept. of Mechanical Engineering, Pennsylvania State Univ., University Park, PA, May 1989.

[7]Greene, W. D., "Triggering of Longitudinal Combustion Instabilities in Rocket Motors," M.S. Thesis, Dept. of Aerospace Engineering, Pennsylvania State Univ., University Park, PA, Dec. 1990.

[8]Ma, Y., Van Moorhem, W. K., and Shorthill, R. W., "Experimental Investigation of Velocity Coupling in Combustion Instability," *Journal of Propulsion and Power*, Vol. 7, No. 5, 1991, pp. 692–699.

Reply by the Authors to V. S. Burnley and F. E. C. Culick

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Introduction

IN Ref. 1, various aspects of the problem of triggered instabilities were addressed using an analytical approach. In Ref. 2, a Comment on Ref. 1, by Burnley and Culick, two issues are raised questioning the validity of this analytical approach. The presentation of the first issue is somewhat confusing because it appears to be resolved by the authors within the comment itself. It is suggested that applying the method of time averaging would have a significant effect on the qualitative behavior of the system, but results are then explicitly cited that indicate that this proposition is erroneous.[3] The second issue raised in the Comment involves the two-mode expansion. Results are cited that suggest that the pulse-triggered behavior found using the two-mode approximation is merely an artifact of that approximation and is not physical.[2] However, these results appear to be incapable of supporting such a general conclusion by themselves, and the method on which the results are based seems to be limited to parametric-type investigations. The two-mode expansion is used in Ref. 1 to allow analytical tractability and to provide physical insight into the mechanism involved in triggered instabilities, such that conclusions of broad scope can be drawn.

Response

The details of our response to the Comment are given in the following three parts.

First, in their Comment on Ref. 1, Burnley and Culick[2] cite results based on the continuation method. This is a technique used in dynamic systems theory for the analysis of systems of ordinary differential equations.[4] The method deals specifically with the steady states of a system, those states for which all of the time derivatives of the system are equal to zero. According to the Hartman–Grobman theorem, the local stability of any steady state can be determined by the stability of the system linearized about that steady-state point. Also, the implicit function theorem provides that the steady states of a system are continuous functions of the system's parameters. The continuation method employs these two theorems to efficiently investigate the behavior of a dynamic system in terms of the system parameters (avoiding explicit solution of the differential system itself). Of particular interest in Ref. 2 are bifurcations, that is, points where qualitative changes in system behavior occur as a parameter (or parameters) is changed. In Ref. 2, the equations governing the flow oscillations are treated with a Galerkin-based method (similar to Ref. 1), resulting in a set of ordinary differential equations that govern the time behavior of the coefficients in the series of acoustic modes used to represent the unsteady motions. In their derivation, the combustion response was represented in the forms from Greene[5] and Kim[6] along with a form due to Baum et al.[7] and Levine and Baum.[8] Upon application of the continuation method for a limited set of system parameters, scenarios were presented in which triggered behavior was observed for two modes, but not for a many-mode, for example, six-mode, model.

Second, the primary concern raised in the Comment regards the two-mode expansion. To apply the Galerkin-based method as in Ref. 1, the unsteady pressure and velocity fields are expressed as an infinite series of the acoustic normal modes. This expansion allows the continuum dynamics of the unsteady flowfield to be treated as a system with discrete states, that is the time-varying mode amplitude coefficients. The resulting discrete system has infinite degrees of freedom, and so, practically, the series must be truncated to a finite number of modes. In Ref. 1, two modes were retained mainly to preserve analytical tractability. This allows global conclusions to be drawn about the behavior of the system, as has been demonstrated in many previous works on nonlinear aspects of combustion instability. For instance, it was possible to rule out several types of combustion response as mechanisms for triggering, allowing attention to be focused on the combustion response functions based on velocity rectification. Without the analytical tractability provided by the two-mode expansion, the analysis would be limited to parametric studies, from which it is often difficult or impossible to extract meaningful conclusions. The two-mode expansion also allows physical insight into the gasdynamic behavior, for example, in terms of mode coupling and intermodal energy transfer.

Third, the primary issue raised in the Comment is that the results in Ref. 1 are limited by the expansion of the nonlinear acoustic field to only the first two acoustic modes. It is suggested that the stable limit cycles found with the two-mode approximation are merely an artifact of that approximation and are not physical. In the Comment, a bifurcation diagram was given that illustrates the failure of the two-mode system in erroneously predicting triggering behavior. In this diagram, however, the first mode's linear growth rate α_1 was the only parameter that was varied. It seems implausible that global conclusions regarding the relevancy of the two-mode approximation could be made by examining the system characteristics only as a function of one parameter. The region of triggerability is likely an n-dimensional region in parametric space, where for the time-averaged, two-mode approximation n equals four (the parameters are the linear growth rate of the first and the second modes, phase difference parameter, and nonlinear combustion response parameter). For the non-time-averaged system with two modes, the linear parameters would form a 32-dimensional space (taking into account the diagonal and off-diagonal linear elements in the source term). Furthermore, this triggerable region may exist for any number of modes and may only be distorted by the two-mode approximation. Perhaps the set of α_1 values in the bifurcation diagram represents a line in the parameter space that is near the edge of the triggerable region; that is, it intersects the region for the two-mode system, but misses it entirely for more modes. Admittedly, this is speculative, and in Ref. 1 no attempt was made to investigate the effects of the two-mode expansion, mainly because the analytical tractability would be compromised. If no triggerable region exists throughout the entire parametric space for a many-mode expansion, then the failure of the two-mode system would be universal and not just

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local. In such case, the triggering results and conclusions drawn from a two-mode analysis would probably be meaningless. However, the Comment does not explain how the parametric results for variation in α_1 justify such a global conclusion.

Conclusion

It is recognized that the problem of triggered instabilities is a complicated one and that it is difficult to analyze the problem from a comprehensive vantage point while at the same time seeking both physical understanding and quantitative accuracy. As such, it is believed that Refs. 1 and 2 represent two studies that each contribute significant progress in researching nonlinear instabilities. As this is a difficult problem involving many uncertainties, both individual methods have their own weakness and merit.

References

[1]Wicker, J. M., Greene, W. D., Kim, S.-I., and Yang, V., "Triggering of Longitudinal Combustion Instabilities in Rocket Motors: Nonlinear Combustion Response," *Journal of Propulsion and Power*, Vol. 12, No. 6, 1996, pp. 1148–1158.

[2]Culick, F. E. C., Burnley, V., and Swenson, G., "Pulsed Instabilities in Solid-Propellant Rockets," *Journal of Propulsion and Power*, Vol. 11, No. 4, 1995, pp. 657–665.

[3]Burnley, V. S., Swenson, G., and Culick, F. E. C., "Pulsed Instabilities in Combustion Chambers," AIAA Paper 95-2430, 1995.

[4]Jahnke, C. C., and Culick, F. E. C., "An Application of Dynamical Systems Theory to Nonlinear Combustion Instabilities," *Journal of Propulsion and Power*, Vol. 10, No. 4, 1994, pp. 508–517.

[5]Greene, W. D., "Triggering of Longitudinal Combustion Instabilities in Rocket Motors," M.S. Thesis, Dept. of Aerospace Engineering, Pennsylvania State Univ., University Park, PA, Dec. 1990.

[6]Kim, S. I., "Nonlinear Pressure Oscillations in Combustion Chambers," Ph.D. Dissertation, Dept. of Mechanical Engineering, Pennsylvania State Univ., University Park, PA, May 1989.

[7]Baum, J. D., Levine, J. N., and Lovine, R. L., "Pulsed Instability in Rocket Motors: A Comparison Between Predictions and Experiments," *Journal of Propulsion and Power*, Vol. 4, No. 4, 1988, pp. 308–316.

[8]Levine, J. N., and Baum, J. D., "A Numerical Study of Nonlinear Instability Phenomena in Solid Rocket Motors," *AIAA Journal*, Vol. 21, No. 4, 1983, pp. 557–564.

Comment on "Shock-Loss Model for Transonic and Supersonic Axial Compressors with Curved Blades"

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A RECENT paper[1] addresses the difficult flow at the entry region to supersonic compressor blades. There are serious errors in some of what is written by Schobeiri about a paper we wrote a few years ago[2] and a flaw in the model he describes in his paper.[1]

We produced an approximate method for the calculation of the stagnation pressure loss and static pressure rise in the inlet region of blades with supersonic inlet velocities. The approximations are appropriate for the type of blades that we have in mind, examples

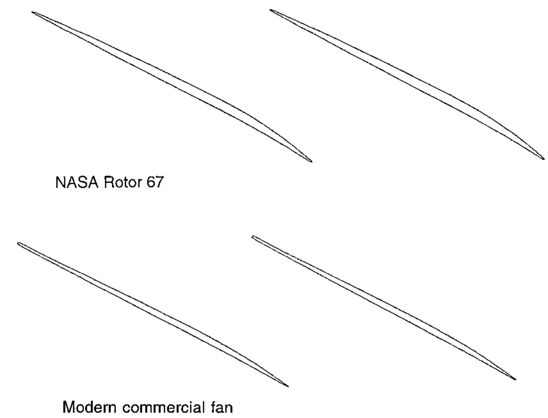

Fig. 1 Two sections through rotors at 90% span.

of which are shown in Fig. 1. Both the blades shown are at the 90% span location; one is the NASA Rotor 67, the other is for a modern high-performance fan for a commercial aircraft. Unlike the blades shown in Schobeiri's paper, the blades are both thin and nearly flat over the forward part. Because of the thinness and flatness, it is a good approximation in the forward part to neglect the force produced by the blades along their direction in computing the pressure and momentum balance in this direction; a momentum balance in any other direction would require the blade force to be explicitly included. Neglecting the thickness of the blades in the momentum equation, particularly at the leading edge, is only an approximation, but as Denton has shown (private communication, 1998), including this has only a very small effect. The thickness of the blades is not, however, neglected in the mass continuity equation because around $M = 1.0$ the flow is exceptionally sensitive to small variations in flow area.

As authors of Ref. 2, we must accept responsibility for Schobeiri misunderstanding what we were doing. It is, however, unfortunate that he has represented his misunderstanding in terms of our ineptitude. For example, he states that considerable discrepancies existed between the left- and right-hand sides of the momentum equation; this is simply untrue. What we did was to plot the left- and right-hand sides of the equation to show that equality could only occur for particular combinations of inlet Mach number M_1 and Mach number M_2 at outlet from the inlet region. Other criticisms Schobeiri makes are either untrue or are part of the approximation that we have reason to believe is realistic.

Before moving on from Schobeiri's misconceptions about Ref. 2, it is worth mentioning that the goal of a simple model like this is to give insight into the main controlling factors or parameters. The requirement is no longer to come up with the most complete description of the flow; now we have three-dimensional computational fluid dynamics (CFD) to do this for us. (In this respect the need is quite different from, for example, that in the 1950s when people such as Levine[3] were trying to give the most complete and accurate description possible for the leading-edge region in supersonic flow.) Simple methods, which isolate a few effects, are useful if they can give us understanding that the more complete description does not. Thus, for example, Ref. 2 showed that when a strong shock is ahead of the leading edge the minimum loss in two dimensions is that of the normal shock, but the loss can be a lot higher if the blade is thick and the incidence increases. The principal weakness of any such method based on two dimensionality is the need to include the variation in streamtube thickness in the spanwise direction. Even small changes can have a very pronounced effect on both pressure rise and loss. The only realistic way to determine these streamtube thickness variations is with three-dimensional CFD.

The flows in many fan geometries have now been calculated by three-dimensional Reynolds-averaged Navier–Stokes methods. Two examples for NASA Rotor 67 are shown in Fig. 2, where contours of Mach number computed using the Denton TIP3D code are shown at the 90% span location. In both cases shown, the rotational speed is the design value, but one case is near peak efficiency and the

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